(7) Q No.→ To find the condition that the line $y = mx + c$ will touch the parabola $y^2 = 4ax$

Ans.→ ∵ the eqn. of parabola is

$$y^2 = 4ax \quad \text{--- (1)}$$

and the eqn. of a line which touch the parabola is

$$y = mx + c \quad \text{--- (2)}$$

Solving (1) and (2), we have

$$(mx+c)^2 = 4ax$$

$$\text{or, } (mx+c)^2 - 4ax = 0$$

$$m^2x^2 + 2mxc + c^2 - 4ax = 0$$

$$m^2x^2 + 2x(mc - 2a) + c^2 = 0 \quad \text{--- (A)}$$

when, $y = mx + c$ will touch the parabola

Hence the roots of eqn. (A) are equal for equal roots dis. $= 0$

$$4(mc - 2a)^2 - 4m^2c^2 = 0$$

$$(mc - 2a)^2 = m^2c^2$$

or, $m^2c^2 - 2\cdot 2mca + (2a)^2 = m^2c^2$

or, $\cancel{m^2c^2} - 4mca + 4a^2 = \cancel{m^2c^2}$

or, $-\cancel{4}mc\cancel{a} = -\cancel{4}\cancel{a^2}a$

$\therefore c = \frac{a}{m}$

$\therefore y = mx + c$ i.e. $y = mx + \frac{a}{m}$ will always touch the parabola $y^2 = 4ax$

—o—

① QN.→ If the line $2x + 3y = 1$ touches the parabola $y^2 = 4ax$, find the length of the latus-rectum.

Ans.→ We know that the length of the latus rectum $= 4a$

$2x + 3y = 1$ — ①

$\therefore y = \frac{1-2x}{3}$

$y^2 = 4ax$ — ②

from ① and ②

$\frac{(1-2x)^2}{9} = 4ax$

or, $(1-2x)^2 = 36ax$

$4x^2 - 4x + 1 - 36ax = 0$

or, $4x^2 - 4x(1+9a) + 1 = 0$ — Ⓐ

$\therefore$ when the line ① touches ② the both roots eqn. Ⓐ are same

i.e. dis. $= 0$

$\cancel{16}(1+9a)^2 = \cancel{4\times4}\times1$

$(1+9a)^2 = 1$

or, $a^2 + 18a + \cancel{1} = \cancel{1}$

$a[81a + 18] = 0$

$\therefore$ Either $a = -\frac{\cancel{18}2}{\cancel{81}9} = -\frac{2}{9}$

$\therefore$ Length of the latus rectum $= 4a$

$= 4 \times -\frac{2}{9} = -\frac{8}{9}$ (magnitude)